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Vector Leptoquarks Could Be Rather Light?

A.V. Kuznetsov and N.V. Mikheev*

Division of Theoretical Physics, Department of Physics,
Yaroslavl State University, Sovietskaya 14,
150000 Yaroslavl, Russian Federation.

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Abstract

Some low-energy manifestations of a minimal extension of the Standard Model based on the quark-lepton $SU(4)_V \otimes SU(2)_L \otimes G_R$ symmetry of the Pati-Salam type are analysed. Given this symmetry a new type of mixing in the quark-lepton interactions is shown to be required. An additional arbitrariness of the mixing parameters could allow to decrease noticeably the lower bound on the leptoquark mass M_X originated from the π and K decays and the μe conversion. The only mixing independent bound emerging from the cosmological limit on the $\pi^0 \to \nu\bar{\nu}$ decay width is $M_X > 18\ TeV$.

*E-mail addresses: phth@cnit.yaroslavl.su, physteo@univ.yars.free.net

Although the Standard Model predictions are in a good agreement with an experiment now, see e.g. ref. [1], a hope for new physics beyond the Standard Model undoubtedly exists. If the high symmetry consecutive restoration with an energy increase is assumed, the questions are pertinent of the next symmetry being restored after the electroweak one and the next mass scale arising after the m_W scale. We should like to draw the attention to the alternative which probably has not been completely studied. It is the Minimal Quark-Lepton Symmetry Model based on the $SU(4)_V$ group with the lepton number as the fourth color [2]

$$\begin{pmatrix} u^1 \\ u^2 \\ u^3 \\ \nu \end{pmatrix}_i , \qquad \begin{pmatrix} d^1 \\ d^2 \\ d^3 \\ \ell \end{pmatrix}_i , \qquad (i = 1, 2, 3), \tag{1}$$

where the i index labels the fermion generations. The following attractive features of this extension of the Standard Model could be specified: (i) a natural explanation for the quark fractional hypercharge, (ii) an absence of the proton decay, and (iii) an indirect evidence that the broken left-right symmetry of the $SU(2)_L \otimes SU(2)_R$ type could exist.

In the recent paper [3] the possible manifestations of an extra Z' boson arising within the quark-lepton symmetry model have been considered. There appears to be more exotic object as the charged and colored gauge X boson named leptoquark which causes the interconversions of quarks and leptons. The tightest restriction on the leptoquark mass M_X is known [4] to be obtained from the analyses of π and K meson rare decays. The estimations of the lower bound on the leptoquark mass were carried out in refs. [5, 6], however, the mixing in the interactions of the leptoquark currents was not taken into account there. It can be shown that such a mixing inevitably occurs in the theory. Really, the mixing in the quark interaction with the W bosons being depicted by the Cabibbo-Kobayashi-Maskawa matrix is sure to exist in Nature. The quark-lepton symmetry necessarily causes the mixing in the leptoquark interaction, say, with $d, s, b-$ quarks and the charged leptons ℓ. On the other hand, it leads at the loop-level to the ultraviolet divergent non-diagonal transitions $\ell \to \ell'$ in vacuum through the quark-leptoquark pair. Consequently, it is necessary for the renormalizability of the model to include all kinds of mixing at the tree-level.

Let us suppose that the Standard Model $SU(3)_c \otimes SU(2)_L \otimes U(1)$ extension is realized on the base of the $SU(4)_V \otimes SU(2)_L \otimes G_R$ group. We assume that the right G_R symmetry will restore at the appreciably higher mass scale than the $SU(4)_V$ one, so the explicit form of the G_R group is not adjusted. It is interesting to note, however, that, if the G_R group is interpreted as the $U(1)_R$ group of the right hypercharge Y_R [3], we have $Y_R = \pm 1$ for the up and down fermions, both quarks and leptons. It is tempting to interpret this fact as the evidence for the right hypercharge to be actually the doubled third component of the right isospin. Hence the G_R group is possible to be the $SU(2)_R$ one. Further we restrict ourselves to the consideration of the semi-simple group $SU(4)_V \otimes SU(2)_L$. Three fermion generations are combined into the {4,2} representations of the type

$$\begin{pmatrix} u^c & d^c \\ \nu & \ell \end{pmatrix}_i, \qquad (i = 1, 2, 3), \tag{2}$$

where c is the color index to be further omitted. In the general case, none of the u, d, ν, ℓ components is the mass eigenstate. Due to the identity of three representations (2) they always could be regrouped so that one of the components was diagonalized with respect to mass. If we diagonalize the charged lepton mass matrix, the representations (2) can be rewritten to the form

$$\begin{pmatrix} u & d \\ \nu & \ell \end{pmatrix}_\ell = \begin{pmatrix} u_e & d_e \\ \nu_e & e \end{pmatrix}, \begin{pmatrix} u_\mu & d_\mu \\ \nu_\mu & \mu \end{pmatrix}, \begin{pmatrix} u_\tau & d_\tau \\ \nu_\tau & \tau \end{pmatrix}, \tag{3}$$

where the indices $\ell = e, \mu, \tau$ correspond to the states which are not the mass eigenstates and are included into the same representations as the charged leptons ℓ

$$\nu_\ell = \mathcal{K}_{\ell i} \nu_i, \quad u_\ell = \mathcal{U}_{\ell p} u_p, \quad d_\ell = \mathcal{D}_{\ell n} d_n. \tag{4}$$

Here ν_i, u_p, and d_n are the mass eigenstates

$$\begin{aligned} \nu_i = (\nu_1, \nu_2, \nu_3), \quad u_p &= (u_1, u_2, u_3), = (u, c, t), \\ d_n &= (d_1, d_2, d_3), = (d, s, b), \end{aligned} \tag{5}$$

and $\mathcal{K}_{\ell i}$, $\mathcal{U}_{\ell p}$, and $\mathcal{D}_{\ell n}$ are the unitary mixing matrices.

The well-known Lagrangian of the interaction of the charged weak currents with the W bosons in our notations has the form

$$\begin{aligned}\mathcal{L}_W &= \frac{g}{2\sqrt{2}}[(\bar{\nu}_\ell O_\alpha \ell) + (\bar{u}_\ell O_\alpha d_\ell)]W^*_\alpha + h.c. = \\ &= \frac{g}{2\sqrt{2}}[\mathcal{K}^*_{\ell i}(\bar{\nu}_i O_\alpha \ell) + \mathcal{U}^*_{\ell p}\, \mathcal{D}_{\ell n}\, (\bar{u}_p O_\alpha d_n)]W^*_\alpha + h.c.,\end{aligned} \tag{6}$$

where g is the $SU(2)_L$ group constant, and $O_\alpha = \gamma_\alpha(1-\gamma_5)$. The standard Cabibbo-Kobayashi- Maskawa matrix is thus seen to be $V = \mathcal{U}^+\mathcal{D}$. This is as far as we know about $\mathcal{U}$ and $\mathcal{D}$ matrices. The $\mathcal{K}$ matrix descriptive of the mixing in the lepton sector has been the object of intensive experimental investigations in recent years.

Subsequent to the spontaneous $SU(4)_V$ symmetry breaking up to $SU(3)_c$ on the M_X scale six massive vector bosons are separated from the 15-plet of the gauge fields to generate three charged and colored leptoquarks. Their interaction with the fermions (5) has the form

$$\mathcal{L}_X = \frac{g_S(M_X)}{\sqrt{2}}[\mathcal{D}_{\ell n}(\bar{\ell}\gamma_\alpha d^c_n) + (\mathcal{K}^+\,\mathcal{U})_{ip}(\bar{\nu}_i\gamma_\alpha u^c_p)]X^c_\alpha + h.c.\,, \tag{7}$$

where the color index $'c'$ is written once again. The constant $g_S(M_X)$ can be expressed in terms of the strong coupling constant α_S at the leptoquark mass scale M_X, $g^2_S(M_X)/4\pi = \alpha_S(M_X)$.

If the momentum transferred is $q \ll M_X$, then the Lagrangian (7) in the second order leads to the effective four-fermion vector-vector interaction of quarks and leptons. By using the Fiertz transformation, lepton-current-to-quark-current terms of the scalar, pseudoscalar, vector and axial-vector types may be separated in the effective Lagrangian. Let us note that the construction of the effective lepton-quark interaction Lagrangian requires taking account of the QCD corrections estimated by the known technique [7, 8]. In our case the leading log approximation $ln(M_X/\mu) \gg 1$ with $\mu \sim 1\, GeV$ to be the typical hadronic scale is quite applicable. Then the QCD correction amounts to the appearance of the magnifying factor $Q(\mu)$ at the scalar and pseudoscalar terms

$$Q(\mu) \;=\; \left(\frac{\alpha_S(\mu)}{\alpha_S(M_X)}\right)^{4/\bar{b}} . \tag{8}$$

Here $\alpha_S(\mu)$ is the effective strong coupling constant at the hadron mass μ scale, $\bar{b} = 11 - \frac{2}{3}\bar{n}_f$, $\bar{n}_f$ is the averaged number of the quark flavors at the scales $\mu^2 \leq q^2 \leq M_X^2$. If the condition $M_X^2 \gg m_t^2$ takes place, then we have $\bar{n}_f \simeq 6$, and $\bar{b} \simeq 7$.

Let us investigate the contribution of the leptoquark interaction (7) to the low-energy processes to establish the bounds on the model parameters from existing experimental limits. As the analysis shows, the tightest restrictions on the leptoquark mass M_X and the mixing matrix $\mathcal{D}$ elements can be obtained from the experimental data on rare π and K decays and $\mu^- \to e^-$ conversion in nuclei. They are represented in the left part of table 1.

In the description of the interactions of π and K mesons it is sufficient to take the scalar and pseudoscalar terms only. As we shall see later, these terms acquire in addition to the QCD corrections an extra enhancement at the amplitude by the small quark current masses. The corresponding part of the effective Lagrangian takes the form

$$\begin{aligned}
\Delta\,\mathcal{L}_{eff} \;&=\; -\frac{2\pi\alpha_S(M_X)}{M_X^2}\,Q(\mu)\,[\mathcal{D}_{\ell n}(\mathcal{U}^{+}\mathcal{K})_{pi}(\bar{\ell}\gamma_5\nu_i)(\bar{u}_p\gamma_5 d_n) + h.c. + \\
&+\; \mathcal{D}_{\ell n}\mathcal{D}^{*}_{\ell' n'}(\bar{\ell}\gamma_5\ell')(\bar{d}_{n'}\gamma_5 d_n) + \\
&+\; (\mathcal{K}^{+}\,\mathcal{U})_{ip}(\mathcal{U}^{+}\,\mathcal{K})_{p'i'}(\bar{\nu}_i\gamma_5\nu_{i'})(\bar{u}_{p'}\gamma_5 u_p) \;-\; (\gamma_5 \to 1)].
\end{aligned} \tag{9}$$

These interactions will contribute to the rare π and K meson decays strongly suppressed in the Standard Model.

One can easily see that the leptoquark contribution to the $\pi \to e\nu$ decay is not suppressed by the electron mass in contrast to the $W-$contribution. The corresponding part of the amplitude could be represented in the form

$$\Delta\mathcal{M}^{X}_{\pi e\nu} \;=\; -\,\frac{2\pi\alpha_S(M_X)}{M_X^2}\,\mathcal{D}_{ed}\,\mathcal{U}^{*}_{\ell u}\,\frac{f_\pi\; m_\pi^2\; Q(\mu)}{m_u(\mu) + m_d(\mu)}\,(\bar{e}\gamma_5\nu_\ell), \tag{10}$$

where $f_\pi \simeq 132\ MeV$ is the $\pi\ell\nu$ decay constant, $m_{u,d}(\mu)$ are the running quark masses at the μ scale. Let us note that the ratio $Q(\mu)/m(\mu)$ is the renormalization group invariant, since the $Q(\mu)$ function (8) determines also

the law of the quark mass running. To the $\mu \simeq 1\ GeV$ scale there correspond the well-known quark current masses $m_u \simeq 4\ MeV, m_d \simeq 7\ MeV$ and $m_s \simeq 150\ MeV$, see e.g. refs. [14, 15]. Taking into account the interference of the amplitude (10) and the known $W-$ exchange amplitude we get the following expression for the decay width ratio $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu) \equiv R$

$$R \;=\; R_W[1 \;-\; \frac{2\sqrt{2}\pi\ \alpha_S(M_X)\ m_\pi^2\ Q}{G_F M_X^2 m_e(m_u+m_d)}\ Re(\frac{\mathcal{D}_{ed}\mathcal{U}_{eu}^*}{V_{ud}})], \tag{11}$$

where $R_W = (1.2352 \pm 0.0005) \cdot 10^{-4}$ is the value of the ratio in the Standard Model [16]. Using the combined results on R of two recent experiments at PSI and TRIUMF [9] we get the following lower bound on the leptoquark mass at the 90 % C.L., see line 1 of table 1. In the paper [5] an attempt was also made to estimate the leptoquark mass from the R ratio, however, an additional assumption was taken there on the $SU(4)$ group constant to be equal to the weak constant g, and the QCD corrections were not considered.

An amplitude of the process $K_L^0 \to e^-\mu^+$ can be found similarly to eq. (10) to be

$$\mathcal{M}_{Ke\mu}^X \;=\; -\frac{\sqrt{2}\pi\alpha_S(M_X)\ f_K\ m_K^2\ Q}{M_X^2\ (m_s+m_d)}\ (\mathcal{D}_{ed}\mathcal{D}_{\mu s}^* \;+\; \mathcal{D}_{es}\mathcal{D}_{\mu d}^*)\ (\bar{e}\gamma_5\mu). \tag{12}$$

where $f_K \simeq 160\ MeV$ is the $K\ell\nu$ decay constant. In place of the estimation of the leptoquark mass bound $M_X > 350\ TeV$ obtained in ref. [6], where the mixing in the leptoquark interaction was not taken into account, we find from the precised experimental data [12] the bound, see line 5 of table 1.

Within recent years the experimental limit on the $Br(K_L^0 \to \mu^+\mu^-)$ value was noticeably lowered to became close to the unitary limit $Br_{abs} = 6.8 \cdot 10^{-9}$. Thus the effective leptoquark contribution to $Br(K_L^0 \to \mu^+\mu^-)$ isn't likely to be larger than $(1 \div 2) \cdot 10^{-9}$. The process amplitude can be easily obtained from eq. (12) by replacing $e \to \mu$.

An amplitude of one more rare K_L^0 decay into electron and positron through the intermediate leptoquark could be also obtained from eq. (12) by replacing $\mu \to e$. The limits on the model parameters in these cases are represented in lines 4,6 of table 1.

Among the charged K^+ meson rare decays which go at the tree level in this model, the tightest limits are established on the decays $K^+ \to \pi^+\mu^-e^+$ [10]

and $K^+ \to \pi^+ \mu^+ e^-$ [11]. An amplitude of the process $K^+ \to \pi^+ \mu^+ e^-$ can be written in the form

$$\mathcal{M}^X_{K\pi\mu e} = -\frac{2\pi\alpha_S(M_X)}{M_X^2} \frac{f^0_+(q^2)\,(m_K^2 - m_\pi^2) + f^0_-(q^2)\,q^2}{m_s - m_d} \, Q \, \mathcal{D}_{ed}\mathcal{D}^*_{\mu s} \, (\bar{e}\mu). \tag{13}$$

Here q is the four-momentum of the lepton pair and $f^0_{+,-}$ are the known form factors of the $K^0_{\ell 3}$ decay. An amplitude of the $K^+ \to \pi^+ \mu^- e^+$ decay can be obtained from eq. (13) by interchanging e and μ. The bounds on the model parameters arising from these decays contain the same matrix elements as from the $K^0_L \to e\mu$ decay but individually, see lines 2,3 of table 1.

A low-energy process under an intensive experimental searches, where the leptoquark could manifest itself is the μe conversion in nuclei. The coherent μe conversion is the most convenient for the observation when the nucleus remains in the ground state, and consequently the electrons are monoenergetic, with the maximum possible energy $\simeq m_\mu$. An effective Lagrangian for the coherent μe conversion contains the scalar and vector quark currents only. In the model under discussion it has the form

$$\Delta \mathcal{L}_{\mu e} = -\frac{2\pi\alpha_S(M_X)}{M_X^2} \mathcal{D}_{ed}\mathcal{D}^*_{\mu d}[\frac{1}{2}(\bar{e}\gamma_\alpha\mu)(\bar{d}\gamma_\alpha d) \; - \; (\bar{e}\mu)(\bar{d}d)Q(\mu)]. \tag{14}$$

Using the calculation technique of ref. [17] we estimate the branching ratio of the μe conversion in titanium for the interaction of the type (14). The result allows us to establish the bound on the model parameters, see line 7 of table 1, on the base of the experimental data [13].

One can see from table 1 that the restrictions on the model parameters contain the elements of the unknown unitary mixing matrices $\mathcal{D}$ and $\mathcal{U}$, which are connected by the condition $\mathcal{U}^+\mathcal{D} = V$ only. Thus the possibility is not excluded, in principle, when the bounds obtained did not restrict M_X at all, e.g. if the elements $\mathcal{D}_{ed}$ and $\mathcal{D}_{\mu d}$ were rather small. It would correspond to the connection of the τ lepton largely with d quark in the $\mathcal{D}$ matrix, and electron and muon with s and b quarks. In general, it is not contradictory to anything even if appears unusual. In this case a leptoquark could give more noticeable contribution to the flavor-changing decays of τ lepton and η, η', Φ and B mesons. However, a relatively poor accuracy of these data doesn't yet allow to restrict the parameters essentially.

We could find only one occasion when the mixing-independent lower bound on the leptoquark mass arises, namely, from the decay $\pi^0 \to \nu\bar{\nu}$. In the paper [18] the cosmological estimation of the width of this decay was found

$$Br(\pi^0 \to \nu\bar{\nu}) < 2.9 \cdot 10^{-13} \ .$$

Within the Standard Model this value is proportional to m_ν^2. The process is also possible through the leptoquark mediation, without the suppression by the smallness of neutrino mass. The process amplitude has the form

$$\mathcal{M}^X_{\pi\nu\nu} = \frac{\pi\alpha_S(M_X)\ f_\pi\ m_\pi^2\ Q}{\sqrt{2}\ M_X^2\, m_u}\ (\mathcal{K}^+\mathcal{U})_{iu}\, (\mathcal{U}^+\mathcal{K})_{uj}\ (\bar{\nu}_i\gamma_5\nu_j), \tag{15}$$

On summation over all neutrino species i, j the decay probability is mixing-independent. As a result the bound on the leptoquark mass occurs

$$M_X > 18\ TeV. \tag{16}$$

In conclusion, we have analysed in detail the experimental data on rare π and K decays and μe conversion and we have found the restrictions on the vector leptoquark mass to contain the elements of an unknown mixing matrix $\mathcal{D}$. The only mixing independent bound (16) arises from the cosmological estimations.

In our opinion, possible experimental manifestations of the considered minimal quark-lepton symmetry model would be an object of further methodical studies. For example, the search of possible leptoquark evidence in the $p\bar{p}$ collider high-energy experiments via the reactions $d\bar{d} \to e^+\mu^-,\ e^-\mu^+$ could be of interest. On the other hand, further searches of flavor-changing decays of τ lepton and η, η', Φ and B mesons are desirable.

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Table 1: The bounds on the leptoquark mass and mixing matrix elements from the experimental limits on the branching ratios of of various processes.

No.	Experimental limit	Ref.	Bound
1	$\dfrac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} = (1.2310 \pm 0.0037) \cdot 10^{-4}$	[9]	$\dfrac{M_X}{\vert Re(\mathcal{D}_{ed}\mathcal{U}^*_{eu}/V_{ud})\vert^{1/2}} > 210\ TeV$
2	$Br(K^+ \to \pi^+\mu^-e^+) < 7 \cdot 10^{-9}$	[10]	$\dfrac{M_X}{\vert\mathcal{D}_{es}\mathcal{D}^*_{\mu d}\vert^{1/2}} > 50\ TeV$
3	$Br(K^+ \to \pi^+\mu^+e^-) < 2.1 \cdot 10^{-10}$	[11]	$\dfrac{M_X}{\vert\mathcal{D}_{ed}\mathcal{D}^*_{\mu s}\vert^{1/2}} > 120\ TeV$
4	$Br(K^0_L \to \mu^+\mu^-) = (7.3 \pm 0.4) \cdot 10^{-9}$	[4]	$\dfrac{M_X}{\vert Re(\mathcal{D}_{\mu d}\mathcal{D}^*_{\mu s})\vert^{1/2}} > 500 \div 600\ TeV$
5	$Br(K^0_L \to \mu e) < 3.3 \cdot 10^{-11}$	[12]	$\dfrac{M_X}{\vert\mathcal{D}_{ed}\mathcal{D}^*_{\mu s} + \mathcal{D}_{es}\mathcal{D}^*_{\mu d}\vert^{1/2}} > 1200\ TeV$
6	$Br(K^0_L \to e^+e^-) < 5.3 \cdot 10^{-11}$	[12]	$\dfrac{M_X}{\vert Re(\mathcal{D}_{ed}\mathcal{D}^*_{es})\vert^{1/2}} > 1400\ TeV$
7	$\dfrac{\Gamma(\mu^-Ti \to e^-Ti)}{\Gamma(\mu^-Ti \to capture)} < 4.6 \cdot 10^{-12}$	[13]	$\dfrac{M_X}{\vert\mathcal{D}_{ed}\mathcal{D}^*_{\mu d}\vert^{1/2}} > 670\ TeV$